

Subject Code: R13102/R13

Set No - 1

I B. Tech I Semester Regular Examinations Jan./Feb. - 2015

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours

Max. Marks: 70

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**

PART-A

- 1.(i) solve the D.E $(1+xy)x dy + (1-xy)y dx = 0$
- (ii) Solve $(D^2+4)y = \sin 2x$
- (iii) Expand $e^x \cos y$ near $(1, \pi/4)$ by Taylors series method
- (iv) Find $L\left(\int_0^t e^{-t} \cos t dt\right)$
- (v) Solve $z(y-x) = qy^2 - px^2$ where $p = \frac{\partial}{\partial x}, q = \frac{\partial}{\partial y}$
- (vi) Solve by method of separation of variables $2x \frac{\partial z}{\partial x} - 3y \frac{\partial z}{\partial y} = 0$

[4+3+4+4+4+3]

PART-B

- 2.(a) Find the equation of the curve satisfying the D.E $(1+x^2)dy + (2xy-4x^2)dx = 0$ and passing through origin
- (b) Find the equation of the system of orthogonal trajectories of the family of the curves $r^n \sin n\theta = a^n$, where 'a' is a parameter.

[8+8]

3.(a) solve the D.E $D^2(D^2+4)y = 320(x^3+2x^2+e^x)$

(b) Solve the D.E $y^{111} + y = \cos(2x-1)$

[8+8]

4.(a) Find the maximum and minimum values of $x^3 - 3xy^2 - 15x^2 - 15y^2 + 72x$

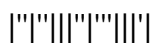
(b) If $z = \log(e^x + e^y)$ show that $\left(\frac{\partial^2 z}{\partial x^2}\right)\left(\frac{\partial^2 z}{\partial y^2}\right) = \frac{\partial^2 z}{\partial x \partial y}$

[8+8]

5.(a) Solve the D.E $(D^2 + 2D + 1)y = 3te^t$ if $y(0) = 4$ & $y'(0) = 2$ using Laplace transform

(b) Using Convolution theorem find $L^{-1}\left(\frac{1}{s^2(1+s)^2}\right)$

[8+8]



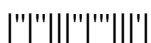
6.(a) Solve the P.D.E $p/x^2 + q/y^2 = z$, where $p = \frac{\partial}{\partial x}, q = \frac{\partial}{\partial y}$

(b) Obtain P.D.E of all spheres whose centre lies on z – axis with a given radius r.

7. An insulated rod of length L has its ends A and B maintained at 0°C and 100°C respectively under steady state conditions prevail .If B is suddenly reduced to 0°C and maintained at 0°C ,find the temperature at a distance x from A at time t

[8+8]

[16]



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PART-A

- 1.(i) Solve the D.E $y(2x^2y + e^x)dx = (y^3 + e^x)dy$
 (ii) Write the Equation of simple harmonic motion and find its solution
 (iii) Find the relation between the functions $u = \frac{x}{y}$ & $v = \frac{x+y}{x-y}$
 (iv) Find $L\left(\frac{e^{-at} - e^{-bt}}{t}\right)$
 (v) Solve the P.D.E $xp - yq = y^2 - x^2$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$
 (vi) Solve the $(D^2 - 2DD')z = e^{2x} + x^3y$ where $D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}$

[4+3+4+4+4+3]

PART -B

- 2.(a) A radioactive substance disintegrates at a rate proportional to it's mass. When it's mass is 10 mgm, the rate of disintegration is 0.051 mgm per day. How long will it take for the mass to be reduced from 10 mgm to 5 mgm.
 (b) Solve $\frac{dy}{dx} = x^3 - 2xy$ if $y(1) = 2$. [8+8]
- 3.(a) solve the D.E $(D^2 - 1)y = x \sin x$
 (b) Solve the D.E $y^{111} - y^{11} - y^1 + y = 1 + x^2$ [8+8]
- 4.(a) if $u = f(r)$ and $x = r \cos \theta, y = r \sin \theta$ then prove that $\left(\frac{\partial^2 u}{\partial x^2}\right) + \left(\frac{\partial^2 u}{\partial y^2}\right) = f''(r) + \frac{1}{r} f'(r)$
 (b) If the sum of three numbers is a constant, then prove that their product is maximum when they are equal. [8+8]

5.(a) Find $L^{-1}\left(\frac{s+1}{s^2(1+s)^2}\right)$

(b) Using Laplace transform, Evaluate $\int_0^{\infty} \frac{e^{-at} \sin^2 t}{t} dt$

[8+8]

6.(a) Form the partial Differential equation by elimination $F(xy+z^2, x+y+z) = 0$

(b) Solve the P.D.E $z(z^2+xy)(p x-q y) = x^4$ where $p = \frac{\partial}{\partial x}, q = \frac{\partial}{\partial y}$

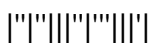
[8+8]

7. A string of length 100 cm is tightly stretched between $x = 0$ and $x = 100$ and is displaced from its equilibrium positions by imparting each of its points an initial velocity given by

$$g(x) = \begin{cases} x & \text{if } 0 \leq x \leq 50 \\ 100-x & \text{if } 50 < x \leq 100 \end{cases}$$

Then find the displacement at any subsequent time.

[16]



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PART-A

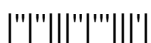
- 1.(i) Solve the D.E $y^2 dx + (-y^2 + x^2 - xy) dy = 0$
 (ii) Solve the D.E $(D^4 + 18D^2 + 81)y = 0$
 (iii) If $x = r \cos \theta$, $y = r \sin \theta$ then prove that $\frac{\partial r}{\partial x} = \frac{\partial x}{\partial r}$ & $\frac{1}{r} \frac{\partial x}{\partial \theta} = r \frac{\partial \theta}{\partial x}$
 (iv) Find $L^{-1} \left\{ \log \left(1 + \frac{1}{s^2} \right) \right\}$
 (v) Solve the P.D.E $p(1+q) = qz$ where $p = \frac{\partial}{\partial x}$, $q = \frac{\partial}{\partial y}$
 (vi) Solve $(D^3 - 3D^2D' + 4D'^2) = e^{x+2y}$ where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$

[4+3+4+4+4+3]

PART -B

- 2.(a) If the air temperature is 20^0c and the body cools for 20 min from 140^0c to 80^0c , find when the temperature will be 35^0c .
 (b) Solve the D.E $dr + (2r \cot \theta + \sin 2\theta) d\theta = 0$ [8+8]
- 3.(a) Solve the D.E $(D^2 + 2D + 1)y = xe^x \cos x$
 (b) Solve the D.E $y^{11} - 2y^1 + y = 1 + e^x$ [8+8]
- 4.(a) Find the dimensions of the rectangular box open at the top of the maximum capacity whose surface area is 108 sq.inches.
 (b) Show that $\log(1 + e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{48} + \dots$ and
 Hence deduce that $\frac{e^x}{1+e^x} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + \dots$ [8+8]
- 5.(a) Find the Laplace transform of Dirac-Delta function
 (b) Solve the D.E $y^{11} + y = 1$ if $y(0) = y^1(0) = y^{11}(0) = 0$ using Laplace transform [8+8]

- 6.(a) Solve the P.D.E $p \cos (x +y) + q \sin (x + y) = z$ where $p = \frac{\partial}{\partial x}, q = \frac{\partial}{\partial y}$
- (b) Obtain P.D.E by eliminating arbitrary functions from $z = f (2x+3y) + g (3x-y)$ [8+8]
7. A square plate is bounded by the lines $x =0, y=0, x =l, y =l$. Its faces are insulated .The temperature of along the upper horizontal edge is given by $u (x, l) = x (l-x)$ when $0 < x < l$ while other edges are kept at zero. Find the steady state temperature. [16]



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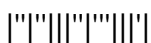
PART-A

- 1.(i) Solve the D.E $xy \, dx - (x^2 + 2y^2)dy = 0$
 (ii) Find the particular integral of $(D^2 + 1) = \sec x$
 (iii) If $u = \frac{y}{z} + \frac{z}{x}$ then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$
 (iv) Find $L[t^2 e^{-2t}]$
 (v) Solve $z^2 = 1 + p^2 + q^2$ where $p = \frac{\partial}{\partial x}, q = \frac{\partial}{\partial y}$
 (vi) Solve by method of separation of variables $4 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 3z$ if $u(0, y) = 3e^{-y} - e^{-5y}$

[4+3+3+4+4+4]

PART -B

- 2.(a) In a certain chemical reaction the rate of change of conversion of a substance at time t is proportional to the quantity of the substance still untransformed at that time. At the end of one hour 60 grams remain and at the end of four hours 21 grams. How many grams of the first substance was there initially.
 (b) Find the orthogonal trajectories of the family of hypocycloids $x^{2/3} + y^{2/3} = a^{2/3}$, where 'a' is a parameter. [8+8]
- 3.(a) Solve the D.E $(D^4 - 1)y = e^x \cos x$
 (b) Solve the D.E $y^{11} - 2y^1 + 2y = 1 + xe^x$ [8+8]
- 4.(a) Find the maximum value of $u = x^2 y^3 z^4$ if $2x + 3y + 4z = a$ by Lagrange's multiplier method.
 (b) If $x = u(1-v)$ and $y = uv$ prove that $\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)} = 1$ [8+8]



5.(a) Solve the D.E $(D^2 + 6D + 9)y = \sin t$ if $y(0) = 1$ & $y'(0) = 0$ using Laplace transform

(b) Find the $L\left(\frac{1 - \cos t}{t^2}\right)$

[8+8]

6.(a) Solve the P.D.E $pqz = p^2(qx + p^2) + q^2(py + q^2)$ where $p = \frac{\partial}{\partial x}$, $q = \frac{\partial}{\partial y}$

(b) Obtain P.D.E by eliminating arbitrary constants a and b from $z = xy + y\sqrt{x+a} + b$

[8+8]

7.(a) Solve $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$, where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$

(b) Find the physically feasible solution of one-dimensional heat flow equation.

[8+8]

